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# Point estimation and interval estimation of mediation effect: multiplicative integration method, Non-parametric Bootstrap and MCMC method

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***Abstract ---****Abstract In view of the problem that the sampling distribution of the mediating effect ab is often not a normal distribution, scholars have proposed three types of methods that do not require any restrictions on the sampling distribution of ab and are suitable for small and medium samples, including the multiplicative integration method, non-parametric Bootstrap and Markov Chain Monte Carlo (MCMC) method. The simulation technique is used to compare the performance of the three methods in the analysis of mediation effect. The results showed that: 1) The MCMC method with prior information has the most accurate ab point estimation; 2) The MCMC method with prior information has the highest statistical power, but it pays the price of underestimating the error rate of type I, and the non-parametric bias correction The centile Bootstrap method is second in terms of statistical power, but it pays the price of overestimating the type I error rate; 3) The MCMC method with prior information has the most accurate estimation of the intermediate effect interval. The results show that when there is prior information, it is recommended to use the MCMC method with prior information; when the prior information is not available, it is recommended to use the deviation-corrected non-parametric percentile Bootstrap method.*

***Type of Paper---*** *Review*

***Keywords*** *: multiplicative integration method; nonparametric Bootstrap method; MCMC method; prior information*

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**Introduction:**

Intervention is a vital methodological concept in social science investigate. In case the autonomous variable X applies a certain impact on the subordinate variable Y through a certain variable M, it is said that M plays a interceding part between X and Y or M is the intervening variable between X and Y (see Figure 1). The reason of interceding impact investigation is to decide whether the relationship between the free variable X and the subordinate variable Y is somewhat or totally inferable to the interceding variable M (Aristocrat & Kenny, 1986; MacKinnon, 2008; Yuan & MacKinnon, 2009). In recent years, mediating effect analysis has been widely used in management (Wood, Goodman, Cook, & Beckman, 2008) and psychology (MacKinnon, Fairchild, & Fritz, 2007; MacKinnon, 2008; Fairchild & McQuillion, 2010) and other disciplines, therefore, the accuracy of mediating effect analysis is particularly important for researchers to correctly understand the relationship between independent variables and dependent variables, as well as the mechanism of this relationship.

At present, the analysis of intermediary effects generally uses the Sobel test method, that is, the estimated value of intermediary effects. $\hat{a}\hat{b}$ (See Figure 1 for the meaning of $\hat{a}$ and $\hat{b}$) Divided by $\hat{a}\hat{b}$

A value greater than the critical z value indicates that the mediation effect is significant, otherwise it indicates that the mediation effect is not significant; or construct a symmetric confidence interval Standard error $\hat{\sigma}_{\hat{a}\hat{b}}$ get a z value ($z = \hat{a}\hat{b}/\hat{\sigma}_{\hat{a}\hat{b}}$), the z value ($\hat{a}\hat{b} - z_{\frac{\alpha}{2}} \times \hat{\sigma}_{\hat{a}\hat{b}}, \hat{a}\hat{b} + z_{\frac{\alpha}{2}} \times \hat{\sigma}_{\hat{a}\hat{b}}$), if the confidence interval does not include, Compared

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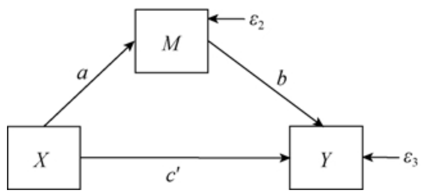
with the critical z value based on the standard normal distribution, if z includes 0, the mediating effect is significant, otherwise, the mediating effect is not significant (MacKinnon, Lockwood, Hoffman, West, & Sheets, 2002; Wen Zhonglin, Zhang Lei, Hou Jietai , Liu Hongyun, 2004). The premise of the Sobel test assumes that the mediating effect $\hat{a}\hat{b}$is a normal distribution and requires a large sample, because only under the normal distribution, can the critical z value based on the standard normal distribution be used. Some researchers have found that even if both $\hat{a}$and $\hat{b}$ are normally distributed, $\hat{a}\hat{b}$is not necessarily a normal distribution. Furthermore, as long as $\hat{a}\hat{b}$ is not zero, the distribution of $\hat{a}\hat{b}$ is skewed, and the peak of the distribution will vary with The value of the mediating effect $\hat{a}\hat{b}$ changes (MacKinnon et al., 2002, 2004, 2008; Cheung & Lau, 2008; Hayes, 2009). Therefore, based on the mediating effect $\hat{a}\hat{b}$ is a normal distribution of the mediating effect analysis method (this article is called For the traditional law) is unreliable.




$$M = \beta_2 + aX + \varepsilon_2 \qquad (1)$$

$$Y = \beta_3 + c'X + bM + \varepsilon_2 \qquad (2)$$

**Figure** 1 Diagram of the mediation effect model

In order to obtain reliable analysis results of the mediation effect, recently, researchers have proposed three types of distributions that do not require $\hat{a}\hat{b}$ from different perspectives.Methods that are subject to any restrictions and are suitable for small and medium samples, namely, the distribution of the Product, the Bootstrap method and the Markov chain Monte Carlo (MCMC) method. So, how do these three types of methods perform in the analysis of mediation effects? At present, the comparison of the three types of methods is carried out by data simulation technology. Mackinnon et al. (2004) simulated and compared the performance of the traditional method, the multiplicative integration method, and five non-parametric Bootstrap methods in the mediation effect analysis, and found that the deviation-corrected non-parametric percentile Bootstrap method provides the most accurate confidence in the mediation effect. Interval, the statistical power is the highest, but it has to pay the price of overestimating the type I error rate under certain conditions. Yuan and MacKinnon (2009) simulated and compared the performance of the multiplicative integration method and the MCMC method in the analysis of the mediation effect, and found that the MCMC method with prior information can reduce the mean square error more effectively than the multiplicative integration method for small and medium samples. , And the MCMC method without prior information and the multiplicative-integral distribution method have the same performance on the mean square error; the 95% confidence interval coverage of the two MCMC methods is greater than the multiplicative-integral distribution method, and the MCMC method with prior information is 95% The confidence interval coverage is significantly greater than 95% (minimum value is 96.3%). However, these two studies have the following problems:

- The three methods are not compared together. In particular, most researchers currently recommend the Bootstrap method for mediating effect analysis (MacKinnon et al., 2004; Pituch, Stapleton, & Kang, 2006; Williams & Mackinnon, 2008; Taylor, MacKinnon, & Tein, 2008; Pituch & Stapleton, 2008; Cheung & Lau, 2008; Biesanz, Falk, & Savalei, 2010), then, between the Bootstrap method and the MCMC method, special Don't be compared with the MCMC method with prior information, whose performance Better? It remains to be studied in depth.
- The comparative indicators of the two studies are insufficient. First, the comparison indicators of the two studies are completely different and lack comparability. Secondly, The three comparison indicators of Mackinnon et al. (2004) are all to examine the confidence interval of the mediation effect, and do not examine the mediation effect point estimation. Third, the 95% confidence interval coverage index used by Yuan and MacKinnon (2009) (if the 95% confidence interval coverage of a method is closer to 95%, the more accurate the interval estimation of the method is) cannot effectively reflect the MCMC method and product The pros and cons of the distribution method in interval estimation. Because the MCMC method constructs a credible interval, a 95% credible interval indicates that the probability of the parameter true value in the reliable interval obtained on the basis of the observation data is 95%, while the multiplicative integration method and the Bootstrap method construct the confidence interval. Interval (confidence interval), 95% confidence interval means that repeated sampling is performed in the population, and a confidence interval is calculated for each sampled sample, then in these confidence intervals, 95% confidence interval

will contain the truth of the parameter (Qiu Haozheng, 2007; Yuan & MacKinnon, 2009; Muthén, 2010), it is obvious that the 95% confidence interval coverage rate will be more conducive to the multiplication integration method and the Bootstrap method. Therefore, Yuan et al. (2009) found that the MCMC method The 95% confidence interval coverage is higher than that of the multiplicative integration method, especially the 95% confidence interval coverage of the MCMC method with prior information is significantly greater than 95%, which is not enough to show that the interval estimation of the MCMC method is inaccurate, so this study The 95% confidence interval coverage rate is not used as a comparison indicator. Fourth, Yuan et al. (2009) also used the multiplicative integration method and the MCMC method to calculate the interval width of a case study of mediation effects (see Table 1), and found that the interval width of the MCMC method with prior information is narrower than that of the MCMC method. Multiplicative integration method; the author further uses the non-parametric Bootstrap method to calculate the interval width of the same data (see Table 1), and finds that the MCMC method with prior information estimates the narrowest interval width (0.089), while the MCMC method without prior information The interval estimation performance of the method, the multiplicative integration method and the non-parametric Bootstrap method is equivalent (0.103~0.110), and the interval estimation of the MCMC method with prior information seems to be the most accurate. So, is the result of this case analysis universal ?

In order to answer this question, this study sets the interval width as a comparison index for the accuracy of interval estimation.In summary, the purpose of this article is to compare the three types of methods together with point estimation and interval estimation. First, briefly introduce the multiplicative integration method, non-parametric Bootstrap and MCMC methods; then use the experimental design of 5 (sample size) 6 (ab combination) 5 (estimation method) to simulate and compare; then discuss the results of simulation comparison and The conclusions are drawn to provide a useful reference for psychology researchers when choosing a mediating effect analysis method.

**Analysis methods of three types of intermediary effects:**

**Multiplicative integration method:**

The basic principle of the multiplicative integration method is that since the mediating effect $\hat{a}\hat{b}$ is not normally distributed in most cases, when constructing the confidence interval of the mediating effect, the critical z value based on the standard normal distribution cannot be used, but the critical z value based on the standard normal distribution should be used. $\hat{a}$ and $\hat{b}$ multiply the critical value of the integral distribution. The default distribution of $\hat{a}\hat{b}$ of the multiplicative distribution method conforms to the multiplicative distribution of two normally distributed random variables (Meeker, Cornwell, & Aroian, 1981), which is an asymmetric and skewed distribution. , The lower confidence limit (confidence limit) requires different critical values, and the resulting confidence interval is an asymmetric confidence interval (MacKinnon et al., 2004; Fritz & MacKinnon, 2007; MacKinnon, Fritz, Williams, & Lockwood, 2007; Tofighi & MacKinnon, 2011; Fang Jie, Zhang Minqiang, Li Xiaopeng, 2011).

**Table** 1 Intermediary effect point estimation and interval estimation for the same measured data by three types of methods

| method | $ab$ point estimation | Lower bound of interval estimation | Upper bound of interval estimation | Interval width |
|---|---|---|---|---|
| Multiplying integration | 0.056 | 0.013 | 0.116 | 0.103 |
| Nonparametric percentile Bootstrap method | 0.056 | 0.008 | 0.118 | 0.110 |
| Non-parametric percentile Bootstrap method for bias correction | 0.056 | 0.014 | 0.123 | 0.109 |
| MCMC method with prior information | 0.051 | 0.013 | 0.102 | 0.089 |
| MCMC method without prior information | 0.056 | 0.011 | 0.118 | 0.107 |

The multiplication and integration method is divided into two steps. First, use formulas (1) and (2) to calculate $\hat{a}$,$\hat{b}$, $\hat{\sigma}_{\hat{a}}$ (standard error of a), and $\hat{\sigma}_{\hat{b}}$ (standard error of b) to obtain the point estimate of the mediation effect a^b^. Second, according to the values of $\hat{a}$,$\hat{b}$, $\hat{\sigma}_{\hat{a}}$, $\hat{\sigma}_{\hat{b}}$ obtained in the first step, as well as $\rho_{ab}$ (correlation of a and b) and the type I error rate $\alpha$ , use PRODCLIN (distribution of the PRODuct Confidence Limits for INdirect effects ) Program (M., Fritz, Williams, & Lockwood, 2007) or the R Mediation software package of R software (Tofighi & MacKinnon, 2011) automatically finds the upper and lower confidence limits in the multiplication and integration table of Meeker et al. (1981) Value, and construct an asymmetric confidence interval ($-\hat{a}\hat{b}$ lower confidence limit standardized critical value $\times \hat{\sigma}_{\hat{a}\hat{b}}$, $+\hat{a}\hat{b}$ upper confidence limit standardized critical value $\times \hat{\sigma}_{\hat{a}\hat{b}}$).

**Non-parametric Bootstrap method:**

Bootstrap method is a kind of resampling methods first proposed by Efron, which can be divided into parameter Bootstrap and non-parametric Bootstrap. This article mainly discusses non-parametric Bootstrap method. The basic idea of the non-parametric Bootstrap method is to treat the original sample as the "population", and through repeated sampling with replacement, the process of extracting a large number of new sub-samples and obtaining statistics, the essence is to simulate the process of randomly extracting a large number of samples from the population. process. Commonly used nonparametric Bootstrap methods include nonparametric percentile Bootstrap method and Bias-corrected nonparametric percentile Bootstrap method. Various statistical software (including SPSS) commonly used at present can perform non-parametric Bootstrap calculations.

**Non-parametric percentile Bootstrap method :**

The non-parametric percentile Bootstrap method is divided into three steps. First, based on the original sample (sample size is n), repeated sampling with replacement is performed while ensuring that the probability of each observation unit being drawn is equal (both 1/n) to obtain a sample A Bootstrap sample with a capacity of n; second, calculate the corresponding mediation effect estimate $\hat{a}\hat{b}$ from the Bootstrap sample obtained in step 1; third, repeat steps 1 and 2 several times (denoted as B, permanent B = 1000), change The average value of the B mediating effect estimates is used as the point estimate of the mediating effect, and the B mediating effect estimates $\hat{a}\hat{b}$ are sorted by numerical value to obtain sequence C, using the 2.5th percentile and 97.5th percentile of sequence C To estimate the 95% confidence interval for the mediation effect (Preacher, Rucker, & Hayes, 2007; Preacher & Hayes, 2008; Hayes, 2009; Fang Jie, Zhang Minqiang, Qiu Haozheng, 2012; Wen Zhonglin, Liu Hongyun, Hou Jietai, 2012).

**Non-parametric percentile Bootstrap method for bias correction:**

The difference-corrected non-parametric percentile Bootstrap method and the non-parametric percentile Bootstrap method have the same intermediate effect point estimates. The deviation-corrected non-parametric percentile Bootstrap method is to correct the non-parametric percentile Bootstrap method. The median value of the default sequence C of the non-parametric percentile Bootstrap method is equal to the intermediate effect estimated value $\hat{a}\hat{b}^*$ obtained from the original sample data. The method is to adjust the percentile point of the confidence interval, which is divided into the following three steps. First, find the percentage ranking of $\hat{a}\hat{b}^*$ in the sequence C, that is, the probability of $\hat{a}\hat{b} < \hat{a}\hat{b}^*$ Φ(z); second, in the standard normal cumulative distribution function, find the corresponding z0 according to Φ(z0) Value; Third, find the probability $\Phi(2Z_0 \quad Z \quad 2)$, corresponding to $2Z_0 \quad Z \quad 2$in the standard normal cumulative distribution function, and use the percentile value of $\Phi(2Z_0 \quad Z \quad 2)$ in the sequence C as The upper and lower confidence limits of the confidence interval, the intermediate effect confidence interval with an estimated confidence of $1-\alpha$ (Fritz & MacKinnon, 2007; MacKinnon, 2008; Preacher & Hayes, 2008; Taylor, MacKinnon, & Tein, 2008; Fang Jie et al. , 2011; Wen Zhonglin et al., 2012).

**Markov chain Monte Carlo (MCMC) method:**

The Markov chain Monte Carlo (MCMC) method is to introduce the Markov chain process into the Monte Carlo simulation under the framework of Bayesian theory to realize the dynamic simulation that the sampling distribution changes with the simulation. The basic idea of MCMC method can be summarized as the following three steps. First, construct a Markov chain to converge to a stationary distribution, which is usually the posterior distribution of the parameter to be tested. In order to make the Markov chain converge to a stable distribution, the previous t iteration values must be discarded. In the analysis of the mediation effect, it is sufficient to take t 1000 times (Yuan & MacKinnon, 2009); second, use the Markov chain to perform Gibb Gibbs sampling, that is, iterative sampling is performed using multiple full conditionals distributions (except for one variable, all other variables are assigned fixed values) to obtain $n - t$ (about 10,000) posterior samples , After getting The test sample is also called the realization value of the Markov chain; third, the posterior sample calculates 10,000 mediation effect estimates $\hat{a}\hat{b}$, and uses the mean value of the 10,000 mediation effect estimates as the point estimate of the mediation effect. Estimated mediation effect $\hat{a}\hat{b}$ sorted by numerical value In order, the 2.5th percentile and the 97.5th percentile are used to obtain a 95% confidence interval estimate of the mediation effect. The calculation of MCMC method is usually completed by WinBUGS, Mplus6, SAS and R software (Mao Shisong, Wang Jinglong, Pu Xiaolong, 2006; Ntzoufras, 2009; Yuan & MacKinnon, 2009; Fang Jie et al., 2011).

The most attractive feature of MCMC is that, according to the Bayesian formula, the posterior distribution of the parameters to be tested is proportional to the product of the prior distribution (probability distribution of prior information) and the likelihood function, so the MCMC method allows researchers to Prior information (prior information) is integrated into the mediation effect analysis, called MCMC (MCMC with informative priors, MCMC inf) method with prior information, which can effectively use other related predecessor research and navigation research resources. In order to obtain more accurate mediating effect analysis results (Muthén, 2010). When the MCMC method with prior information is used to analyze the mediation effect, the prior distributions of the regression coefficients a and b of formulas (1) and (2) are often set to the normal distribution, and the prior distributions of the residuals $\varepsilon_2$ and $\varepsilon_3$ It is often set to an inverse gamma distribution (to ensure that the variance of the residual is positive). Taking regression coefficient a as an example, the prior distribution of a can be expressed as $a \sim N\ (\mu, \tau)$, N means normal distribution, $\mu$ means mean, $\tau$ means accuracy, that is, the reciprocal of variance $\sigma^2$. If the prior distribution is set to $a \sim N\ (0, 10^{-6})$, the mean of the prior distribution is 0 and the variance is large (106), which can be regarded as no prior information, called no prior MCMC (MCMC with non-informative priors, MCMC non-inf) method of verifying information.

**Results and analysis:**

**Bias in the estimation of the intermediate effect point:**

For the same simulation data, the deviations of the intermediate effect point estimates of the multiplicative integration method, the non-parametric Bootstrap method and the MCMC method are calculated respectively. The results are shown in Table 2. Since the estimated values of the intermediate effect points of the non-parametric percentile Bootstrap method and the non-parametric percentile Bootstrap method of bias correction are the same, they are uniformly represented by Boot in Table 2. MCMC*inf* and MCMC*noninf* in Table 2 respectively represent the MCMC method with prior information and the MCMC method without prior information.

The 4×5×2 analysis of variance (ANOVA) is used to analyze the influence of 4 methods, 5 sample sizes, and 2 mediation effects (mediation effect is 0 and not 0) on the bias of the mediation effect point estimation. The analysis of variance showed that only the interaction between the sample size and the method was significant, F(12, 80)=1.964, p=0.039, 2 η p =0.228. Simple main effect analysis shows that when the sample size is 25, the main effect of the method is significant, F(3, 80)= 3.666, p=0.016. Multiple comparisons (using the Bonferroni correction method, the same below) found that the deviation (3.5) of the MCMC method with prior information and the multiplicative integration method (−32.2) are significantly different; when the sample size is 50, the main effect of the method is significant, and F(3, 80)=4.483, p=0.006. Multiple comparisons show that the MCMC method (−23.1) without prior information is Integral distribution method (17.1) has significant difference; under other sample size conditions, the main effect of the method is not significant. Only under the multiplicative integration method, the main effect of the sample size is significant, F (4, 80)=5.528, p=0.001, and multiple comparisons found that the sample size of 25 is significantly different from the sample size of 50, 100, and 1000. When the sample size is 25 The deviation is the largest; under the other methods, the main effect of the sample size is not significant.

For the absolute value of the deviation of the intermediate effect point estimate, the 4×5×2 analysis of variance also shows that only the interaction between the sample size and the method is significant, F(12, 80)=2.04, *p*=0.031, 2 *ηp* = 0.234. Simple main effects analysis shows that when the sample size is 25, the main effect of the method is significant, F(3,80)=6.47, p=0.001. Multiple comparisons found that the deviation absolute value (5.8) of the MCMC method with prior information is significantly smaller than the multiplicative integration method (34.9) and the non-parametric Bootstrap method (40.7); the main effects of the method are not significant under the other sample size conditions. Under the multiplicative integration method, the main effect of the sample size is significant, F(4,80)=4.354, p=0.003, and multiple comparisons found that the sample size of 25 is significantly different from the sample size of 100 and 1000. When the sample size is 25, the deviation is absolute The value is the largest; under the non-parametric Bootstrap method, the main effect of the sample size is significant, $F(4,80)= 6.987$, $p<0.001$, multiple comparisons found that the sample size of 25 is significantly different from the sample size of 50, 200, and 1000. At 25, the absolute value of the deviation is the largest. Under the two MCMC methods, the main effect of sample size was not found to be significant.

**The relative mean square error of the intermediate effect point estimate:**

For the same simulated data, the relative mean square errors of the intermediate effect point estimates of the multiplicative integration method, the non-parametric Bootstrap method and the MCMC method are calculated respectively. The results are shown in Table 2. The 4×5×2 analysis of variance is used to analyze the influence of 4 methods, 5 sample sizes, and 2 mediation effects (mediation effect is 0 and not 0) on the relative mean square error of the mediation effect point estimate. The analysis of variance showed that only the interaction between sample size and method was significant, $F(12, 80)=11.18$, $p<0.001$, 2 $\eta p = 0.626$. Simple main effects analysis shows that the main effects of the method are significant under the five sample size conditions. Multiple comparisons consistently found that the relative mean square error of the MCMC method with prior information is significantly smaller than that of the other three methods, which is similar to Yuan et al. ( 2009) found that the relative mean square error of the MCMC method with prior information is significantly smaller than the results of the multiplicative integration method. Only under the condition of the MCMC method with prior information, the main effect of the sample size is significant, $F(4,80)=42.875$, $p<0.001$, multiple comparisons found that, except for the relative mean square error (2.4) of the sample size of 25 and the sample size The relative mean square error of 50 (7.8), the relative mean square error of the sample size of 50 and the relative mean square error of the sample size of 100 (19.5) are not significantly different, and the relative mean square errors of the remaining sample sizes are all significantly different. As the sample size increases, the relative mean square error also increases.

**Table** 2 The mediating effect bias (×10000) and relative mean square error (×100%) estimated by the three types of methods

| method | *a* *b* 0 | *a* 0.39, *b* 0 | *a* 0, *b* 0.59 | *a* *b* 0.14 | *a* *b* 0.39 | *a* *b* 0.59 | *a* *b* 0 | *a* 0.39, *b* 0 | *a* 0, *b* 0.59 | *a* *b* 0.14 | *a* *b* 0.39 | *a* *b* 0.59 | *a* *b* 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Bias | RMSE | Bias | RMSE | Bias | RMSE | Bias | RMSE | Bias | RMSE | Bias | RMSE | Bias |
| Multiplicative integration | 32.9 | 100.0 | 29.7 | 100.0 | 100.0 | 7.9 | 100.0 | 55.1 | 100.0 | 11.3 | 100.0 | 72.4 | 100.0 |
| Boot | 3.5 | 110.9 | 39.3 | 97.1 | 107.5 | 55.0 | 106.4 | 10.6 | 116.7 | 99.0 | 107.5 | 36.9 | 99.7 |
| MCMCinf | 0.5 | 0.1 | 0.7 | 2.5 | 3.2 | 9.9 | 3.2 | 4.6 | 1.6 | 12.7 | 3.2 | 6.4 | 3.4 |
| MCMCnoninf | 2.6 | 101.1 | 15.9 | 107.2 | 109.6 | 3.0 | 99.0 | 3.0 | 95.6 | 8.0 | 109.6 | 76.2 | 91.1 |
| | | | | | | | N = 50 | | | | | | |
| Multiplicative integration | 5.2 | 100.0 | 29.5 | 100.0 | 100.0 | 23.0 | 100.0 | 10.3 | 100.0 | 44.0 | 100.0 | 9.2 | 100.0 |
| Boot | 4.4 | 95.8 | 0.2 | 101.7 | 107.5 | 3.1 | 108.6 | 16.8 | 83.2 | 4.4 | 100.3 | 21.8 | 109.6 |
| MCMCinf | 0.9 | 1.0 | 6.8 | 8.7 | 3.2 | 21.5 | 9.9 | 1.4 | 6.6 | 2.2 | 9.5 | 1.8 | 10.9 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MCMCnoninf | 6.0 | 85.7 | 30.8 | 95.9 | 109.6 | 43.2 | 100.4 | 4.3 | 97.2 | 28.4 | 85.0 | 46.2 | 117.1 |
| | | | | | | | N = 100 | | | | | | |
| Multiplicative integration | 1.4 | 100.0 | 1.2 | 100.0 | 100.0 | 12.5 | 100.0 | 6.1 | 100.0 | 1.4 | 100.0 | 16.5 | 100.0 |
| Boot | 1.4 | 87.7 | 0.9 | 105.9 | 107.5 | 50.2 | 92.9 | 5.8 | 107.8 | 30.8 | 102.4 | 51.5 | 101.1 |
| MCMCinf | 1.6 | 5.5 | 6.0 | 23.5 | 3.2 | 13.7 | 9.9 | 0.2 | 20.6 | 9.2 | 22.4 | 27.8 | 21.7 |
| MCMCnoninf | 3.5 | 77.2 | 14.8 | 102.4 | 109.6 | 15.2 | 100.4 | -8.8 | 96.1 | 29.2 | 89.6 | 17.1 | 104.1 |
| | | | | | | | N = 200 | | | | | | |
| Multiplicative integration | 2.1 | 100.0 | 3.9 | 100.0 | 100.0 | 23.5 | 100.0 | 5.0 | 100.0 | 18.2 | 100.0 | 12.0 | 100.0 |
| Boot | 0.5 | 90.2 | 4.2 | 116.2 | 107.5 | 1.8 | 99.2 | 4.4 | 88.1 | 2.2 | 97.0 | 6.8 | 111.7 |
| MCMCinf | 0.2 | 17.5 | 10.3 | 46.1 | 3.2 | 3.4 | 41.7 | 1.5 | 42.3 | 5.6 | 45.6 | 8.4 | 45.6 |
| MCMCnoninf | 1.3 | 80.0 | 4.6 | 107.6 | 109.6 | 9.1 | 92.7 | 9.1 | 93.6 | 0.8 | 101.0 | 7.2 | 108.8 |
| | | | | | | | N = 1000 | | | | | | |
| Multiplicative integration | 0.3 | 100.0 | 3.0 | 100.0 | 100.0 | 2.3 | 100.0 | 2.1 | 100.0 | 2.6 | 100.0 | 1.5 | 100.0 |
| Boot | 0.1 | 110.0 | 1.0 | 94.4 | 107.5 | 0.8 | 100.6 | 1.4 | 92.2 | 0.7 | 106.7 | 4.7 | 92.5 |
| MCMCinf | 0.1 | 69.0 | 3.9 | 76.6 | 3.2 | 11.5 | 86.7 | 1.2 | 79.1 | 5.2 | 89.6 | 1.7 | 75.3 |
| MCMCnoninf | 0.0 | 86.8 | 2.8 | 96.5 | 109.6 | 4.7 | 109.9 | 0.2 | 101.9 | 5.7 | 107.6 | 3.5 | 97.1 |

**Statistical power of mediating effect analysis:**

For the same simulation data, the statistical powers of the mediation effect analysis of the multiplicative integration method, the non-parametric Bootstrap method and the MCMC method are calculated respectively. The results are shown in Table 3. In Table 3, P_Boot represents the non-parametric percentile Bootstrap method, Bc_Boot represents the non-parametric percentile Bootstrap method of deviation correction, and MCMC*inf* and MCMC*noninf* respectively represent the MCMC method with prior information and the MCMC method without prior information.

The 5×5 analysis of variance is used to analyze the influence of 5 methods and 5 sample sizes on the statistical power of the mediation effect analysis. The analysis of variance showed that only the main effect of the sample size was significant. As the sample increased, the statistical power increased, $F(4, 50) = 5.646$, $p = 0.001$, $\eta_p^2 = 0.311$. The multiple comparison results show that the statistical power (0.997) of the sample size of 1000 is significantly greater than the sample size of 25(0.345) 50(0.548). The 5×3 analysis of variance is used to analyze the influence of 5 methods and 3 mediating effects on the statistical power of the mediating effect analysis. The analysis of variance showed that only the main effect of the mediating effect was significant, $F(2, 60)=28.521$, $p<0.001$, 2 η p=0.487. The multiple comparison results show that the statistical power (0.288) with a small mediation effect (0.0196) is significantly less than the statistical power (0.790) with a medium mediation effect (0.1521) and the statistical power (0.925) with a large mediation effect (0.3481). In summary, the sample size and the size of the mediation effect are the main determinants of the statistical power of the mediation effect.

Although the two analysis of variance did not find that the main effect of the method is significant, the statistical power of the MCMC method with prior information is always greater than that of the other four methods (see Table 3), that is, the MCMC method with prior information has the highest statistical power. Since previous simulation studies did not include the MCMC method to participate in the comparison, the Bootstrap method of bias correction was considered to have the highest statistical power (MacKinnon et al., 2004; Fritz & MacKinnon, 2007; Pituch et al., 2006, 2008; Williams & Mackinnon, 2008; Taylor et al., 2008). The results in Table 3 also show that the statistical power of the deviation-corrected non-parametric percentile Bootstrap method is always greater than that of the multiplicative integration method, the non-parametric percentile Bootstrap method and the MCMC method without prior information.

**Type I error rate of mediating effect analysis:**

Error in Table 3 represents the type I error rate. The 5×5 analysis of variance is used to analyze the influence of 5 methods and 5 sample sizes on the type I error rate of the mediation effect analysis. The analysis of variance showed that only the main effect of the method was significant, $F(4, 50)=4.145$, $p=0.006$, 2 ηp = 0.249. Multiple comparisons found that the type I error rate of the MCMC method with prior information (0.009) was significantly lower than the deviation-corrected non-parametric percentile Bootstrap method (0.049), and there was no significant difference in the type I error rate among the remaining methods. . The 3×5 analysis of variance is used to analyze the effects of 3 combinations of a and b, and 5 sample sizes on the type I error rate of the mediation effect analysis. The analysis of variance showed that only the main effect of the ab combination was significant, $F(2, 60)=37.936$, $p<0.001$, 2 ηp =0.558, and multiple comparisons found that the type I error rate (0.003) of the combination of 0 and 0 was significantly less than 0.39 The combination of 0 and 0 (0.041), the combination of 0 and 0.59 (0.048), when a=b=0, the type I error rate (maximum 0.009) of various methods is much lower than the true value of 0.05, and also lower than The situation where a and b are not equal to 0 at the same time (see Table 3), which is consistent with the results of previous studies (MacKinnon et al., 2004; Pituch et al., 2006, 2008; Williams & Mackinnon, 2008; Taylor et al. , 2008). Neither analysis of variance found that the main effect of sample size was significant. Therefore, the combination of method and ab is the main factor influencing the type I error rate.

**Table** 3 The statistical power of the mediating effect estimated by the three types of methods, the type I error rate and the 95% interval

| Method | *a* *b* 0 | | *a* 0.39, *b* 0 | | *a* 0, *b* 0.59 | | *a* *b* 0.14 | | *a* *b* 0.39 | | *a* *b* 0.59 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | error | width | error | width | error | width | power | width | power | width | power | width |
| | | | | | N=25 | | | | | | | |
| Multiplicative integration | **0.001** | 0.237 | 0.035 | 0.393 | 0.053 | 0.526 | 0.010 | 0.287 | 0.209 | 0.504 | 0.611 | 0.713 |
| P_Boot | **0.004** | 0.312 | 0.028 | 0.455 | 0.056 | 0.582 | 0.022 | 0.351 | 0.154 | 0.531 | 0.548 | 0.762 |
| Bc_Boot | **0.009** | 0.329 | 0.049 | 0.470 | **0.088** | 0.593 | 0.030 | 0.367 | 0.234 | 0.551 | 0.661 | 0.782 |
| MCMCinf | **0.000** | 0.040 | **0.000** | 0.146 | **0.000** | 0.210 | 0.040 | 0.079 | 1.000 | 0.197 | 1.000 | 0.298 |
| MCMCnoninf | **0.00** | 0.32 | **0.01** | 0.45 | **0.01** | 0.58 | 0.005 | 0.37 | 0.145 | 0.571 | 0.534 | 0.79 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1** | 6 | **5** | 5 | **9** | 5 | | 0 | | | | 0 |
| | | | | | | N=50 | | | | | | |
| Multiplicative integration | **0.006** | 0.108 | 0.054 | 0.242 | 0.051 | 0.348 | 0.032 | 0.157 | 0.605 | 0.332 | 0.959 | 0.483 |
| P_Boot | **0.001** | 0.144 | 0.041 | 0.260 | 0.058 | 0.362 | 0.012 | 0.182 | 0.525 | 0.334 | 0.932 | 0.494 |
| Bc_Boot | **0.004** | 0.151 | 0.070 | 0.265 | **0.089** | 0.366 | 0.049 | 0.190 | 0.643 | 0.346 | 0.951 | 0.503 |
| MCMCinf | **0.005** | 0.037 | **0.000** | 0.130 | **0.000** | 0.192 | 0.058 | 0.070 | 1.000 | 0.177 | 1.000 | 0.266 |
| MCMCnoninf | **0.001** | 0.150 | 0.032 | 0.262 | 0.041 | 0.367 | 0.019 | 0.181 | 0.505 | 0.339 | 0.933 | 0.492 |
| | | | | | | N=100 | | | | | | |
| Multiplicative integration | **0.002** | 0.053 | 0.050 | 0.166 | 0.060 | 0.242 | 0.087 | 0.092 | 0.941 | 0.224 | 1.000 | 0.333 |
| P_Boot | **0.003** | 0.070 | 0.049 | 0.168 | 0.053 | 0.240 | 0.060 | 0.103 | 0.912 | 0.228 | 1.000 | 0.332 |
| Bc_Boot | **0.005** | 0.074 | **0.087** | 0.170 | 0.068 | 0.241 | 0.119 | 0.108 | 0.946 | 0.234 | 1.000 | 0.336 |
| MCMCinf | **0.001** | 0.030 | **0.006** | 0.113 | **0.006** | 0.166 | 0.183 | 0.060 | 1.000 | 0.155 | 1.000 | 0.234 |
| MCMCnoninf | **0.000** | 0.072 | 0.040 | 0.172 | 0.052 | 0.244 | 0.060 | 0.102 | 0.938 | 0.229 | 1.000 | 0.338 |
| | | | | | | N=200 | | | | | | |
| Multiplicative integration | **0.005** | 0.027 | 0.043 | 0.112 | 0.053 | 0.166 | 0.288 | 0.060 | 1.000 | 0.154 | 1.000 | 0.234 |
| P_Boot | **0.003** | 0.036 | 0.055 | 0.114 | 0.060 | 0.165 | 0.222 | 0.063 | 0.999 | 0.156 | 1.000 | 0.233 |
| Bc_Boot | **0.009** | 0.038 | **0.079** | 0.114 | 0.064 | 0.165 | 0.312 | 0.066 | 0.999 | 0.158 | 1.000 | 0.235 |
| MCMCinf | **0.001** | 0.021 | **0.012** | 0.092 | **0.016** | 0.134 | 0.469 | 0.048 | 1.000 | 0.126 | 1.000 | 0.191 |
| MCMCnoninf | **0.003** | 0.036 | 0.057 | 0.116 | 0.041 | 0.166 | 0.203 | 0.063 | 0.999 | 0.158 | 1.000 | 0.235 |
| | | | | | | N=1000 | | | | | | |
| Multiplicative integration | **0.002** | 0.005 | 0.049 | 0.049 | 0.044 | 0.073 | 0.985 | 0.025 | 1.000 | 0.069 | 1.000 | 0.104 |
| P_Boot | **0.002** | 0.007 | 0.046 | 0.049 | 0.053 | 0.073 | 0.99 | 0.025 | 1.000 | 0.068 | 1.000 | 0.103 |
| Bc_Boot | **0.005** | 0.008 | 0.049 | 0.049 | 0.058 | 0.073 | 0.996 | 0.026 | 1.000 | 0.069 | 1.000 | 0.104 |
| MCMCinf | **0.000** | 0.006 | 0.035 | 0.046 | 0.048 | 0.070 | 0.995 | 0.024 | 1.000 | 0.065 | 1.000 | 0.099 |
| MCMCnoninf | **0.000** | 0.007 | 0.044 | 0.049 | 0.066 | 0.073 | 0.982 | 0.025 | 1.000 | 0.068 | 1.000 | 0.103 |

It is worth noting that when a or b is 0, the type I error rate exceeding the upper limit of Bradley's (1978) reasonable variation range of 0.075 all occurs in the deviation-corrected non-parametric percentile Bootstrap method; type I All cases where the error rate is lower than the lower limit of the reasonable variation range of 0.025 in Bradley (1978) occur in the MCMC method, especially in the MCMC method with prior information (see Table 3). This shows that when a or b is 0, the deviation-corrected non-parametric percentile Bootstrap method will overestimate the type I error rate, while the MCMC method with prior information will underestimate the type I error rate.

**Interval width of mediating effect analysis:**

The width in Table 3 represents the width of the interval. The analysis of variance of 5×5×2 is used to analyze the influence of 5 methods, 5 sample sizes, and 2 mediating effect conditions (mediating effect is 0 and not 0) on the width of the analysis of mediating effect. The analysis of variance showed that the main effect of whether the mediating effect is 0 is significant, $F(1, 100)=11.563$, $p=0.001$, 2 η p = 0.104, and the interval width (0.179) where the mediating effect is 0 is significantly smaller than the mediating effect is not 0 The width of the interval (0.239). The interaction between sample size and method is significant, $F(16, 100)=1.859$, $p=0.033$, 2 ηp = 0.229. Simple main effects analysis shows that only when the sample size is 25, the main effect of the method is significant, $F(4, 100)=11.458$, $p<0.001$, multiple comparisons found that the MCMC method with prior information estimates the interval width (0.162) Significantly smaller than the other four methods, the difference between the other four methods is not significant; under the condition of the MCMC method with prior information, the main effect of the sample size is not significant, $F(4, 100)=0.923$, $p= 0.454$, the other methods Under the conditions, the main effect of the sample size is significant. Multiple comparisons found that the interval width with a sample size of 25 is significantly larger than that of other sample sizes, and the interval width with a sample size of 50 is significantly larger than the interval width with a sample size of 1000.

**Conclusion:**

- In terms of mediating effect point estimation, the MCMC method with prior information performs best, the relative mean square error is the smallest under all sample size conditions, and the absolute value of the deviation under the condition of a small sample (n=25) It is also the smallest, and there is no significant difference between the other methods.
- The statistical power of the MCMC method with prior information is the highest, but it pays the price of underestimating the error rate of type I, and the statistical power of the non-parametric percentile Bootstrap method with deviation correction is second, but it pays to overestimate the first type The cost of class error rate.
- In the estimation of the intermediate effect interval, the MCMC method with prior information has the smallest interval width and the best performance.

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